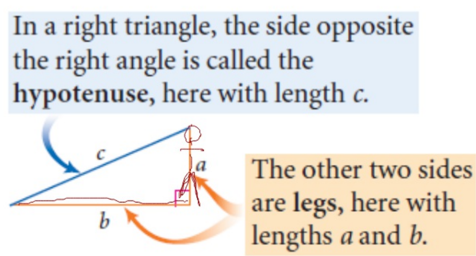
The Pythagorean Theorem

03/13/18

IWBAT understand the Pythagorean Theorem more deeply.

The Pythagorean Theorem

Recall the parts of a triangle.



There is a special relationship between the lengths of the legs and the length of the hypotenuse. This relationship is known today as the Pythagorean Theorem. A theorem is a conjecture that has been proved.

The Pythagorean Theorem

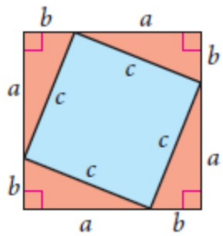
In a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse. If a and b are the lengths of the legs, and c is the length of the hypotenuse, then $a^2+b^2=c^2$.

IWBAT understand the Pythagorean Theorem more deeply.

The Pythagorean Theorem

You are going to perform the Investigate on DG p. 462.

Paragraph Proof: The Pythagorean Theorem



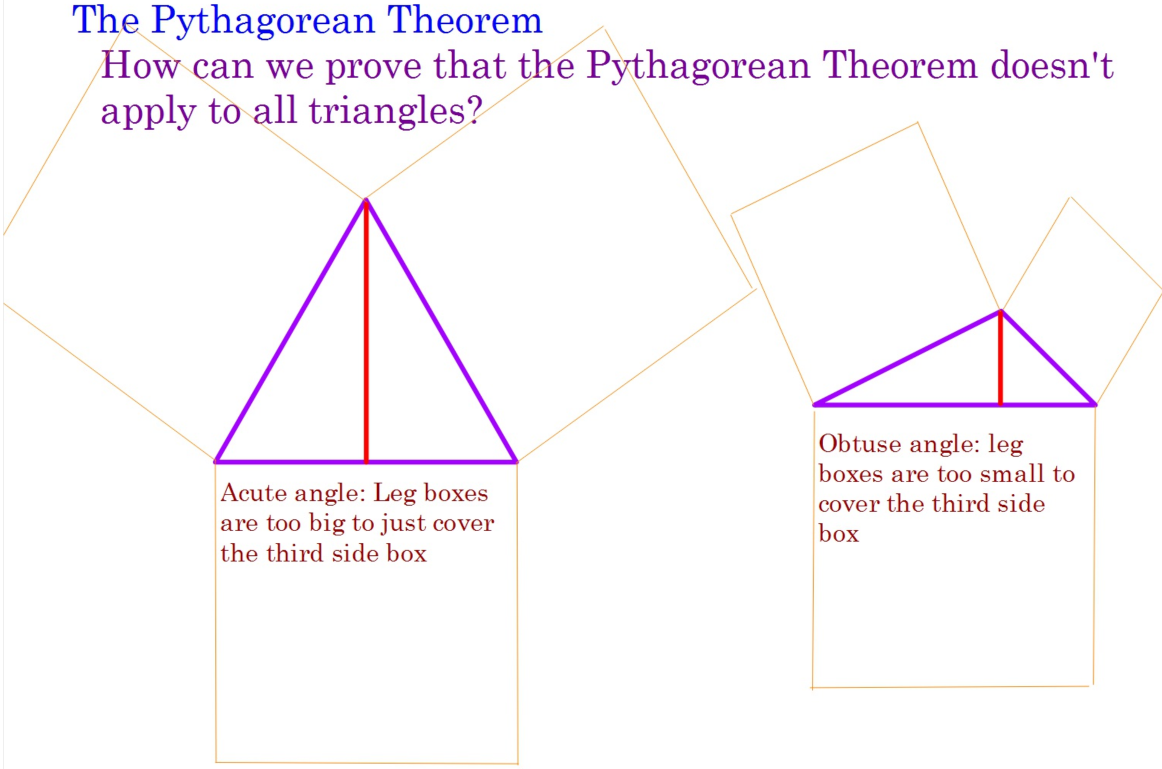
You need to show that $a^2 + b^2$ equals c^2 for the right triangles in the figure at left. The area of the entire square is $(a + b)^2$ or $a^2 + 2ab + b^2$. The area of any triangle is $\left(\frac{1}{2}\right)ab$, so the sum of the areas of the four triangles is $2ab$. The area of the quadrilateral in the center is $(a^2 + 2ab + b^2) - 2ab$, or $a^2 + b^2$.

If the quadrilateral in the center is a square then its area also equals c^2. You now need to show that it is a square. You know that all the sides have length c, but you also need to show that the angles are right angles. The two acute angles in the right triangle, along with any angle of the quadrilateral, add up to 180°. The acute angles in a right triangle add up to 90°. Therefore the quadrilateral angle measures 90° and the quadrilateral is a square. If it is a square with side length c, then its area is c^2. So, $a^2 + b^2 = c^2$, which proves the Pythagorean Theorem. ■

IWBAT understand the Pythagorean Theorem more deeply.

The Pythagorean Theorem

How can we prove that the Pythagorean Theorem doesn't apply to all triangles?



IWBAT understand the Pythagorean Theorem more deeply.

The Pythagorean Theorem

Exercises DG p. 465 #2-16 evens

IWBAT understand the Pythagorean Theorem more deeply.

The Converse of the Pythagorean Theorem

03/15/18

IWBAT discover the converse of the Pythagorean Theorem

The Converse of the Pythagorean Theorem

Three positive integers that work in the Pythagorean equation are called Pythagorean triples. For example, 8-15-17 is a Pythagorean triple because $8^2 + 15^2 = 17^2$.

Here are nine sets of Pythagorean triples.

3-4-5	5-12-13	7-24-25	8-15-17
6-8-10	10-24-26		16-30-34
9-12-15			
12-16-20			

- Choose any of the black triples to measure out in centimeters on your paper as line segments. Construct a triangle from this set of segments. What is the largest angle?
- Repeat with another triple.
- State your results as a conjecture.

IWBAT discover the converse of the Pythagorean Theorem

The Converse of the Pythagorean Theorem

Converse of the Pythagorean Theorem

If the lengths of the three sides of a triangle satisfy the Pythagorean equation, then the triangle is a right triangle.

IWBAT discover the converse of the Pythagorean Theorem

The Converse of the Pythagorean Theorem

Proof: Converse of the Pythagorean Theorem

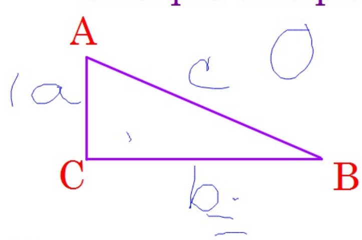
Conjecture: If the lengths of the three sides of a triangle work in the Pythagorean equation, then the triangle is a right triangle.

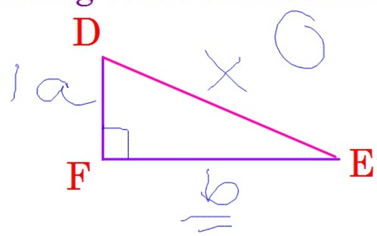
Given: a, b, c are the lengths of the sides of ΔABC and $a^2 + b^2 = c^2$

Show: ΔABC is a right triangle

Plan: Begin by constructing a second triangle, right triangle DEF (with $\angle F$ a right angle), with legs of lengths a and b and hypotenuse of length x. The plan is to show that x = c, so that the triangles are congruent. Then show that $\angle C$ and $\angle F$ are congruent. Once you show that $\angle C$ is a right angle, then ΔABC is a right triangle and the proof is complete.

Critique this plan in writing to be turned in.





IWBAT discover the converse of the Pythagorean Theorem

The Converse of the Pythagorean Theorem

Exercises DG pp. 470-471 #2-14 evens

IWBAT discover the converse of the Pythagorean Theorem

Radical Expressions 03/16/18

IWBAT simplify and multiply square roots and radical expressions.

Radical Expressions 03/16/18

Simplify $\sqrt[2]{50}$ $5\sqrt{2}$

5 10

5 2

Simplify $\sqrt{76}$ $2\sqrt{19}$

2 38

2 19

IWBAT simplify and multiply square roots and radical expressions.

$\sqrt{1801}$ Prime #

$\sqrt{118}$ 2 59

$\sqrt{1222}$ 2 611

$\sqrt{2018}$ 2 1009

$\sqrt{20}$ $2\sqrt{5}$ 2 10 2 5

$\sqrt{405}$ $9\sqrt{5}$ 5 81 9 9

$\sqrt[3]{405}$ $3\sqrt[3]{15}$ 5 81 9 9 3 3 3 3 15 27

$\sqrt[4]{81}$ 3

$\sqrt[4]{405}$ $3\sqrt[4]{5}$

Radical Expressions

Multiply $3\sqrt{6}\times 5\sqrt{2}$ $15\sqrt{12}$ 6 2 3 2 $30\sqrt{3}$

Multiply $7\sqrt{3}\times 2\sqrt{5}$ $14\sqrt{15}$ 3 5

$3\sqrt{6}\times 7\sqrt{6}$

$21\sqrt{36}$ 6 6

126

IWBAT simplify and multiply square roots and radical expressions.

Radical Expressions

Exercises DG p. 474 #1, 3, 5, 6, 12, 18

IWBAT simplify and multiply square roots and radical expressions.

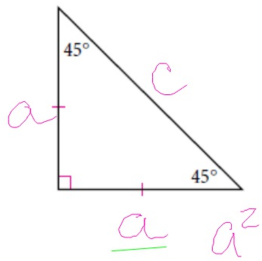
Two Special Right Triangles

03/19/18

IWBAT

- discover the relationships among the lengths of the sides of a 45-45-90 triangle and a 30-60-90 triangle, and
- practice simplifying square roots.

Two Special Right Triangles



What kind of triangle is this?

isosceles right triangle

What is the relationship between the legs and the hypotenuse?

$a^2+a^2=c^2$

$\sqrt{2a^2}=\sqrt{c^2}$

$\sqrt{2}a=c$

$a=5,\ c=5\sqrt{2}$

$5^2+5^2=c^2$

$25+25=c^2$

$\sqrt{50}=\sqrt{c^2}$

10 5

2 5

$=c$

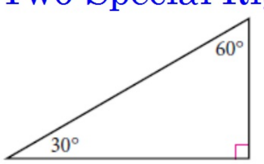
$5\sqrt{2}$

Isosceles Right Triangle Conjecture

In an isosceles right triangle, if the legs have length l, then the hypotenuse has length $l\sqrt{2}$.

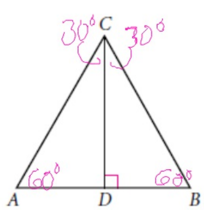
IWBAT discover the relationships among the lengths of the sides of a 45-45-90 triangle and a 30-60-90 triangle, and practice simplifying square roots.

Two Special Right Triangles



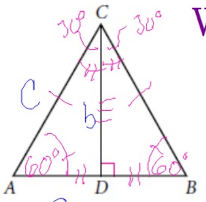
A 30°-60°-90° triangle

If you fold an equilateral triangle along one of its altitudes, the triangles you get are 30°-60°-90° triangles. A 30°-60°-90° triangle is half an equilateral triangle, so it also shows up often in mathematics and engineering.



IWBAT discover the relationships among the lengths of the sides of a 45-45-90 triangle and a 30-60-90 triangle, and practice simplifying square roots.

Two Special Right Triangles



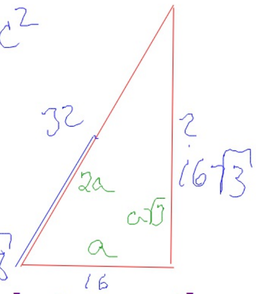
What do we know about this triangle?

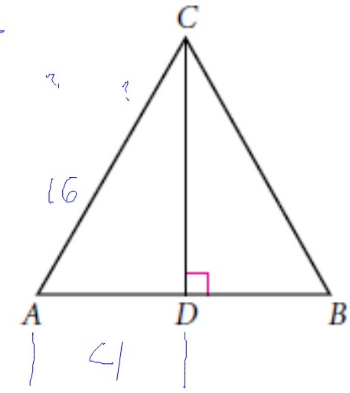
$\triangle ACD \cong \triangle BCD$ ASA

$a^2+b^2=c^2$

$16^2+b^2=32^2$
$-16^2 \quad -16^2$

$b^2 = \sqrt{32^2-16^2} = \sqrt{768}$



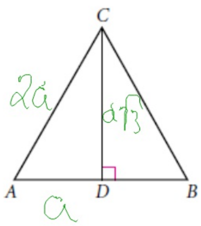


What is the relationship between the legs and the hypotenuse?

$b=\sqrt{768}$

2 · 384
2 · 192
2 · 96
2 · 48
2 · 24
6 · 4
$2 \cdot 2 \cdot 2 \cdot 2$

$16\sqrt{3}$



IWBAT discover the relationships among the lengths of the sides of a 45-45-90 triangle and a 30-60-90 triangle, and practice simplifying square roots.

Two Special Right Triangles

30°-60°-90° Triangle Conjecture

In a 30°-60°-90° triangle, if the shorter leg has length a, then the longer leg has length $a\sqrt{3}$ and the hypotenuse has length $2a$.

Use the Pythagorean Theorem to prove that this relationship is true for any 30°-60°-90° triangle.

$$a^2+(a\sqrt{3})^2=(2a)^2$$

$$a^2 1 + a^2 3 = a^2 4$$

$$1a^2+3a^2=4a^2 \checkmark$$

IWBAT discover the relationships among the lengths of the sides of a 45-45-90 triangle and a 30-60-90 triangle, and practice simplifying square roots.

Two Special Right Triangles

Exercises DG p. 477-478 #1-11 all

7) $12^2 + (12\sqrt{2})^2 = d^2$

IWBAT discover the relationships among the lengths of the sides of a 45-45-90 triangle and a 30-60-90 triangle, and practice simplifying square roots.

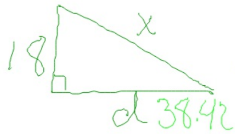
Story Problems

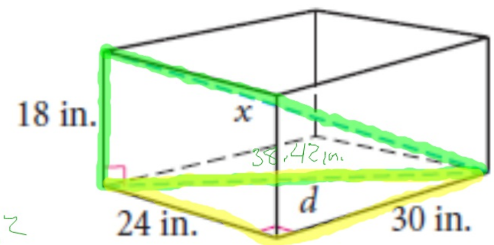
03/20/18

IWBAT apply the Pythagorean Theorem and its converse to story problems.

Story Problems

What is the longest stick that will fit inside a 24-by-30-by-18-inch box?





$A^2+b^2=c^2$

$24^2+30^2=d^2$

$576+900=d^2$

$1476=d^2$

$38.42 \text{ in} = d$

$18^2+38.42^2=x^2$

$324+1476=x^2$

$1800=x^2$

$10\sqrt{18}$

$30\sqrt{2}=x$

42.43 in

$24^2+30^2=d^2 \quad d^2+18^2=x^2$

Substitute

$24^2+30^2+18^2=x^2$

IWBAT apply the Pythagorean Theorem and its converse to story problems.

Story Problems

Exercises DG p. 482-483 #1-5 all

IWBAT apply the Pythagorean Theorem and its converse to story problems.

Distance in Coordinate Geometry

04/03/18

IWBAT discover the Pythagorean relationship on a coordinate plane, derive the equation of a circle from the distance formula, and use the distance formula to solve problems.

Distance in Coordinate Geometry

How far is it from S to V?

$a=5$
$b=5$

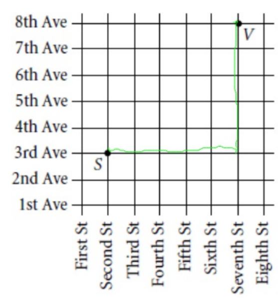
$5^2+5^2=c^2$

$25+25=c^2$

$\sqrt{50}=\sqrt{c^2}$

$\sqrt{5\cdot5\cdot2}=5\sqrt{2}=c$

$7\approx c$



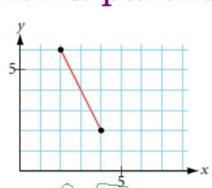
$\sqrt{25\cdot10}$

$\sqrt{5^2\cdot10}$

$5\sqrt{10}$

With a partner, perform Investigation 1 on pp. 486-487.

a.



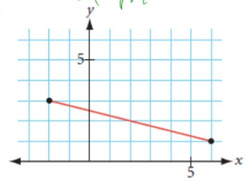
$2\sqrt{5}$

b.

$2\sqrt{10}$

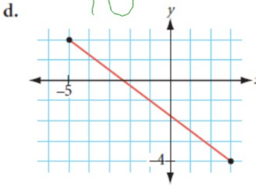
c.

$2\sqrt{17}$



d.

10



IWBAT discover the Pythagorean relationship on a coordinate plane, derive the equation of a circle from the distance formula, and use the distance formula to solve problems.

Distance in Coordinate Geometry

What if the points are too far apart to plot? How will you find the distance between them then? (x, y)

$\sqrt{\Delta x^2 + \Delta y^2} = d$

$(-28, -78)\ (642, 94)$

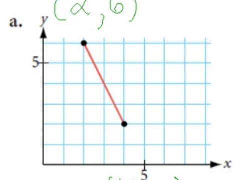
$$\boxed{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = d}$$

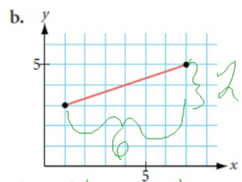
$\sqrt{(642 - {}^-28)^2 + (94 - {}^-78)^2} = c$

$\sqrt{670^2 + 172^2} = c$

$c = 691.73$

Find the distances between the points in a. and b. using only the locations of the end points.





$\sqrt{(4-2)^2 + (2-6)^2} = c$

$\sqrt{2^2 + (-4)^2}$

$\sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

$\sqrt{(1-7)^2 + (3-5)^2}$

$\sqrt{(-6)^2 + (-2)^2}$

$\sqrt{36 + 4}$

$\sqrt{40}$ $\quad 2\sqrt{10}$

$4 \cdot 10$

$2 \cdot 2 \cdot 2 \cdot 5$

IWBAT discover the Pythagorean relationship on a coordinate plane, derive the equation of a circle from the distance formula, and use the distance formula to solve problems.

Distance in Coordinate Geometry

Distance Formula

The distance between points A(x_1, y_1) and B(x_2, y_2) is given by

$$(AB)^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

$$\text{or} \quad AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Find the distance between A (8, 15) and B (-7, 23).

$x_1\ y_1 \qquad x_2\ y_2$

$\sqrt{(8 - {}^-7)^2 + (15 - 23)^2}$ $\quad 17$

$\sqrt{(15)^2 + (-8)^2}$

$\sqrt{225 + 64}$

$\sqrt{289}$

IWBAT discover the Pythagorean relationship on a coordinate plane, derive the equation of a circle from the distance formula, and use the distance formula to solve problems.

Distance in Coordinate Geometry

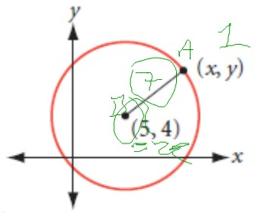
Write the equation for the circle with the center (5, 4) and radius 7.

$(AB)^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$7^2 = (x-5)^2 + (y-4)^2$

$0 = (x-5)^2 + (y-4)^2 - 49$

$r^2 = (x - x_c)^2 + (y - y_c)^2$



Complete Investigate 2 on p. 488.

a. Center = (1, −2), r = 8   b. Center = (0, 2), r = 6

c. Center = (−3, −4), r = 10

a. $8^2 = (x-1)^2 + (y-(-2))^2$

$6^2 = (x-0)^2 + (y-2)^2$

$6^2 = x^2 + (y-2)^2$

IWBAT discover the Pythagorean relationship on a coordinate plane, derive the equation of a circle from the distance formula, and use the distance formula to solve problems.

Distance in Coordinate Geometry

Equation of a Circle

The equation of a circle with radius r and center (h, k) is $(x - \underline{h})^2 + (y - \underline{k})^2 = \underline{r}^2$.

$x^2 + y^2 = r^2$ (at the origin)

Exercises p. 489 #1-10

IWBAT discover the Pythagorean relationship on a coordinate plane, derive the equation of a circle from the distance formula, and use the distance formula to solve problems.

Perimeter & Area via the Distance Formula

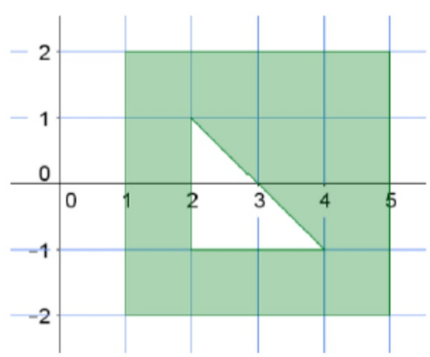
04/05/18

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

Perimeter & Area via the Distance Formula

Find the area of the shaded region.

a.



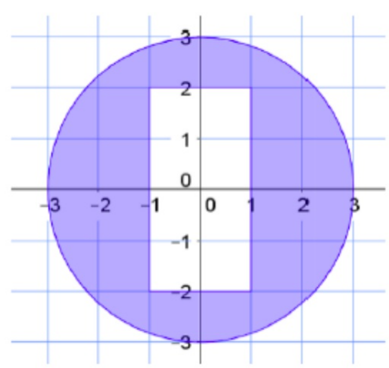
$A_\Delta = \frac{b \cdot h}{2}$

$4 \times 4 = 16$

$\frac{2 \times 2}{2} = 2 \; -$

14

b.



$A_\circ = \pi r^2$ $A_\circ = \pi (3)^2$

$A_\square = b \cdot h$ $= 9\pi$

$A_\square = 4 \times 2 = 8$

$9\pi - 8 \approx 20$

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

Perimeter & Area via the Distance Formula

Consider a triangular region in the plane with vertices O(0,0), A(5,2), and B(3,4). What is the perimeter of the triangular region? What is the area of the triangular region?

$\overline{OA}$ $OA = \sqrt{(0-5)^2 + (0-2)^2} = \sqrt{29}$

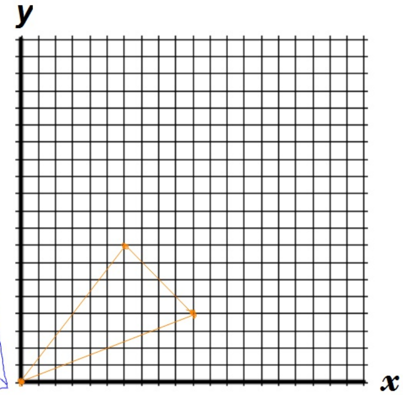
$\overline{OB}$ $OB = \sqrt{(0-3)^2 + (0-4)^2} = 5$

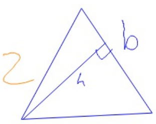
$\overline{AB}$

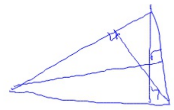
$AB = \sqrt{(5-3)^2 + (2-4)^2} = 2\sqrt{2}$

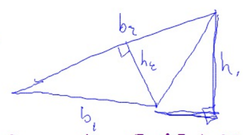
$P = 5 + 2\sqrt{2} + \sqrt{29}$

$P \sim 5 + 2.8 + 4.4 = 12.2$









Process 1: find the area using A=(b*h)/2 and an altitude.

Process 2: find the area using a different method.

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

Perimeter & Area via the Distance Formula

Process 1: find the area using A=(b*h)/2 and an altitude. $\frac{38 \times 17}{2} = 19 \times 17 = 323\,mm^2$

$m\overline{OA} = \frac{2}{5} \therefore m\overline{BC} = -\frac{5}{2}$

$\overline{OA}$ $y = \frac{2}{5}x$ $\overline{BC}$ $y - 4 = -\frac{5}{2}(x-3)$

$y = -\frac{5}{2}(x-3) + 4$

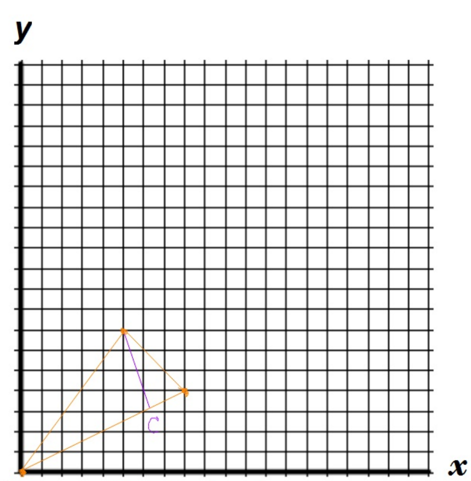
$\frac{2}{5}x = -\frac{5}{2}(x-3) + 4$ $y = \frac{2}{5}\left(\frac{155}{29}\right) = \frac{310}{145}$

$\frac{2}{5}x = -\frac{5}{2}x + \frac{15}{2} + 4$ $C\left(\frac{151}{29}, \frac{302}{145}\right)$

$\left(\frac{2}{5} + \frac{5}{2}\right)x = 11.5$

$\frac{29}{10}x = \frac{31}{2}$

$x = \frac{31}{2} \cdot \frac{10}{29} = \frac{155}{29}$



(0,0) (3,4) (5,2)

O B A

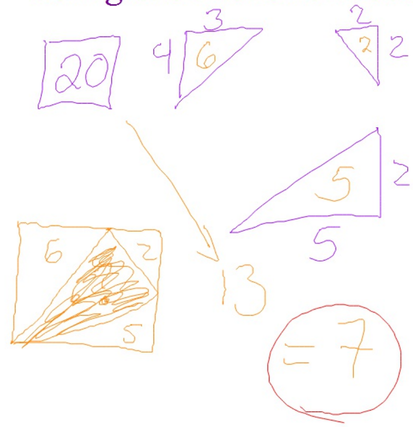
$BC = \sqrt{\left(\frac{155}{29} - 3\right)^2 + \left(\frac{310}{145} - 4\right)^2} \sim 2.99$

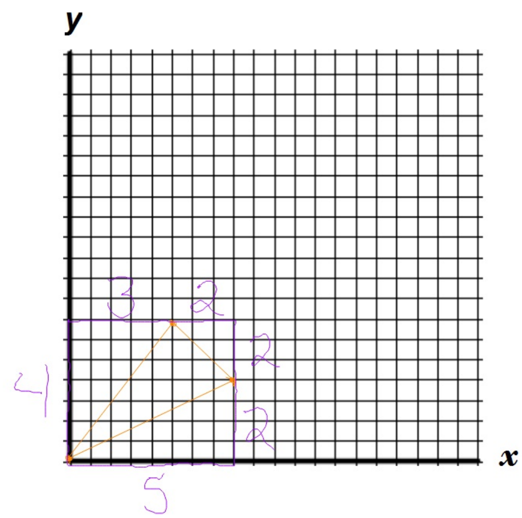
$A_{\triangle ABC} \sim 2.99 \times \sqrt{29} \times \frac{1}{2} \sim 1.50\sqrt{29} \sim$ 8.05 ~~7.87~~

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

Perimeter & Area via the Distance Formula

Process 2: find the area using a different method.

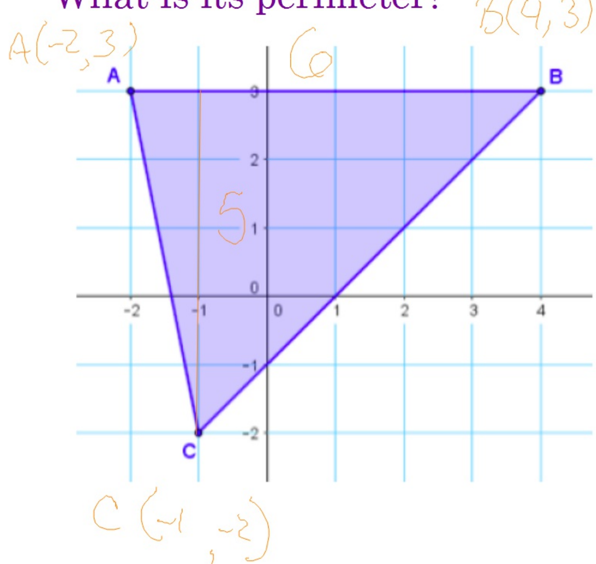




IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

Perimeter & Area via the Distance Formula

Given the triangle below, what is its area? What is its perimeter?



$A = \frac{5 \times 6}{2} = 5 \times 3 = 15$

$AC \quad \sqrt{(-2-{-1})^2+(3-{-2})^2} = \sqrt{26}$

$+BC \quad \sqrt{(4-{-1})^2+(3-{-2})^2} = 5\sqrt{2}$

$+\ 6$

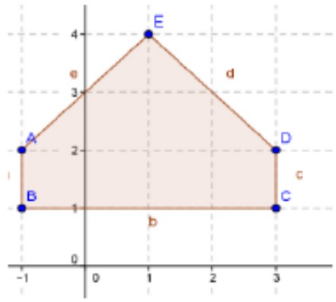
$P = 6 + 5\sqrt{2} + \sqrt{26}$

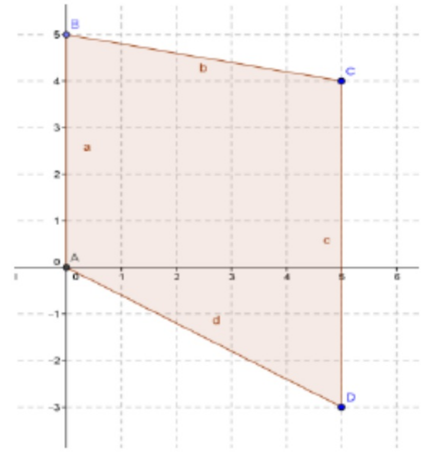
$6 + 7 + 5.1 \sim 18.1$

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

Perimeter & Area via the Distance Formula

Find the area of these polygons.

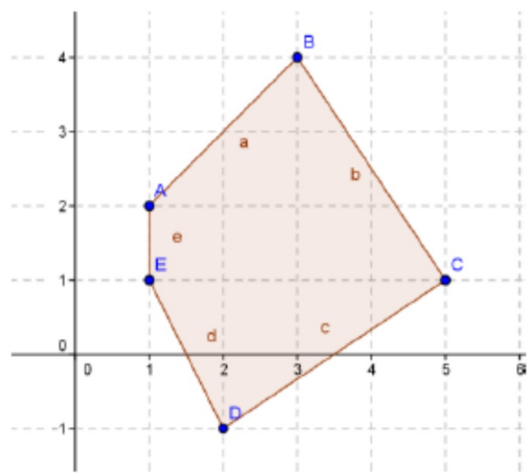




IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

Perimeter & Area via the Distance Formula

Challenge: Find the area and perimeter of this polygon.



IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

Perimeter & Area via the Distance Formula

Find the perimeter and area of the polygons in exercises p.370-371 #1-3 and area of #4 & 5.

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

What should be in your notebook as of today:

Exercises DG p. 465 #2-16 evens

Exercises DG pp. 470-471 #2-14 evens

Exercises DG p. 474 #1, 3, 5, 6, 12, 18

Exercises DG p. 477-478 #1-11 all

Exercises DG p. 482-483 #1-5 all

Exercises p. 489 #1-10

Exercises p.370-371 #1-3 and #4 & 5

Completing the square 04/09/18

IWBAT complete the square to convert an equation for a circle into standard form in order to be able to identify the center and radius of that circle.

Completing the square

$$x^2 + 6x - 3 = 0$$

$ax^2 + bx + c$

$(x^2 + 6x + 9) - 3 = 0 + 9$

$c = \left(\frac{b}{2}\right)^2$

FOIL

$(x+3)^2 - 3 = 9$

$+3 \quad +3$

$(x+3)^2 = 12$

$(x+3)(x+3)$

$x^2 + 3x + 3x + 9$

$x^2 + 6x + 9$

IWBAT complete the square to convert an equation for a circle into standard form in order to be able to identify the center and radius of that circle.

Completing the square

$$x^2 + 18x + y^2 - 18y = 0$$

$ax^2 + bx + c = 0$

if $c = \left(\frac{b}{2}\right)^2$, $a = 1$

perfect square

$(x^2 + 18x + \underline{81}) + (y^2 - 18y + \underline{81}) = 0$

$+81$

$+81$

$(x + 9)^2 + (y - 9)^2 = 162$

$(x-h)^2 + (y-k)^2 = r^2$

(h, k) center of circle

$$\boxed{\begin{array}{c}(x+9)^2 + (y-9)^2 = 162 \\ (-9, 9) \quad r = \sqrt{162} = 9\sqrt{2}\end{array}}$$

IWBAT complete the square to convert an equation for a circle into standard form in order to be able to identify the center and radius of that circle.

Completing the square

$$x^2 + y^2 + 20x - 26y + 253 = 0$$

$x^2 + 20x + y^2 - 26y + 253 = 0$

$(x^2 + 20x + \underline{100}) + (y^2 - 26y + \underline{169}) + 253 = 0$

$+100$

$+169$

$= 269$

$(x+10)^2 + (y-13)^2 + 253 = 269$

$-253 \quad 253$

$= 16$

$(-10, 13)$

$r = 4$

IWBAT complete the square to convert an equation for a circle into standard form in order to be able to identify the center and radius of that circle.

$$x^2 - 30x + y^2 - 16x + 84 = 0$$

$$(x^2 - 30x + \underline{225}) + (y^2 - 16x + \underline{64}) + 84 = 0$$

$$(x - 15)^2 + (y - 8)^2 = 205$$

$$\begin{array}{r} 225 \\ 64 \\ \hline = 289 \\ -84 \\ 205 \end{array}$$

$(15, 8)$ $r = \sqrt{205}$

$$x^2 + 24x - y^2 + 8y + 49 = 0$$

$$x^2 + 24x - y^2 + 8y + 49 = 0$$

$$(x^2 + 24 + \underline{144}) - (y^2 - 8y + \underline{16}) + 49 = 0$$

$$(x + 12)^2 - (y - 4)^2 + 49 = 160$$

$$-49 \quad -49$$

$$\begin{array}{r} 144 \\ 16 \\ \hline 160 \end{array}$$

$$(x + 12)^2 - (y - 4)^2 = 111$$

hyperbola $(-12, 4)$ $a = \sqrt{111}$

Pp. 496-498
#1, 2, 4, 7, 8, 13, 14, 19, 25